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Structure preserving model reduction of port-Hamiltonian systems by moment matching at infinity[☆]

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ABSTRACT

Model reduction of port-Hamiltonian systems by means of the Krylov methods is considered, aiming at port-Hamiltonian structure preservation. It is shown how to employ the Arnoldi method for model reduction in a particular coordinate system in order to preserve not only a specific number of the Markov parameters but also the port-Hamiltonian structure for the reduced order model. Furthermore it is shown how the Lanczos method can be applied in a structure preserving manner to a subclass of port-Hamiltonian systems which is characterized by an algebraic condition. In fact, for the same subclass of port-Hamiltonian systems the Arnoldi method and the Lanczos method turn out to be equivalent in the sense of producing reduced order port-Hamiltonian models with the same transfer function.

1. Introduction

The port-Hamiltonian approach to modeling and control of complex physical systems has emerged as a systematic and unifying framework during the last twenty years; see Ortega, van der Schaft, Mareels and Maschke (2001); van der Schaft (2000a,b); van der Schaft and Maschke (1995). The port-Hamiltonian modeling captures the physical properties of the considered system including the energy dissipation, stability and passivity properties as well as the presence of conservation laws. Another important issue the port-Hamiltonian approach deals with is the interconnection of the physical system with other physical systems creating the so-called physical network. In real applications the dimensions of such interconnected port-Hamiltonian state-space systems rapidly grow both for lumped- and (spatially discretized) distributed-parameter models. Therefore an important issue concerns *model reduction* of these high-dimensional models for further analysis and control.

The goal of this work is to investigate model reduction methods which preserve the *port-Hamiltonian structure*, and, as a consequence, *passivity*.

In the systems and control literature there are a variety of methods and techniques used for model reduction serving different purposes. Passivity preserving model reduction is considered in Antoulas (2005b), Ionutiu, Rommes and Antoulas (2008) and Sorensen (2005). Reduced order models of mechanical systems preserving the Lagrangian structure of the original system are presented in Lall, Krysl and Marsden (2003). Standard balancing techniques go back to Moore (1981). Recent applications of balancing to port-Hamiltonian systems are given in Hartmann (2009), Hartmann, Vulcanov and Schütte (in press) and Polyuga and van der Schaft (2008, in press). For an overview of model reduction techniques we refer the reader to Antoulas (2005a) and Schilders, van der Vorst and Rommes (2008).

The so-called *moment matching* methods are an important class of model reduction methods which are based on the notion of *moment* of a transfer function of a linear system Antoulas (2005a). The idea behind the moment matching approach is to equalize a specific number of the leading coefficients of the Laurent series expansion of the transfer function of the full order model with that of the reduced order model at certain points in the complex plane. The *Partial realization* problem is solved when the expansion is considered around infinity and is of special interest in this paper. The *Padé approximation* is a problem of the moment matching at zero. In the general case, the moment matching problem is known as *rational interpolation*. There is a

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vast literature on this topic; see the book Komzsik (2003), the lecture notes Van Dooren (1995) as well as Gallivan, Grimme and Van Dooren (1999), Gragg and Lindquist (1983), Grimme (1997), Grimme, Gallivan and Van Dooren (1998), Gugercin, Antoulas and Beattie (2008), Gutknecht (1994), Mehrmann and Xu (2000), etc., discussing different approaches, drawbacks and advantages, and numerical issues along with the use of the Arnoldi and Lanczos procedures which are also of interest in this paper.

Consider a linear, single-input, single-output, continuous-time system Σ described by equations of the form

$$\begin{cases} \dot{x} = Ax + bu, \\ y = cx, \end{cases} \tag{1}$$

with the state-space vector $x(t) \in \mathbb{R}^n$, input $u(t) \in \mathbb{R}$, output $y(t) \in \mathbb{R}$, and constant matrices $A \in \mathbb{R}^{n\times n}$, $b \in \mathbb{R}^n$, $c \in \mathbb{R}^{1\times n}$. The transfer function of the system (1) can be approximated by the finite sum (using expansion around infinity):

$$G(s) = c(sI - A)^{-1}b \approx \sum_{i=0}^{M} cA^i b \cdot s^{-i},$$

where the coefficients cA^ib are called the first moments at infinity or the *Markov parameters*. If k is the dimension of the reduced order system then the constant M is equal to $k-1$ in case of the Arnoldi and to $2k-1$ in case of the Lanczos procedures. Thus the Lanczos method matches twice as many Markov parameters as the Arnoldi method. This expansion shows that matching moments at infinity will make the behavior of the reduced order model approximate that of the full order model well for large frequencies in the frequency domain and for small times in the time domain.

The utility of the Krylov methods comes from the fact that the projection maps used in these model reduction procedures can be generated with only inner-products and matrix-vector multiplications. If A matrix turns out to be sparse, one can compute the projection maps relatively cheaply, which is of big advantage for large-scale dynamical systems, when the order n is greater than 1000. Moreover, since the computation of moments is quite often ill-conditioned, the Krylov methods play an important role again, since they yield matching of the moments without explicitly computing them.

In this paper we concentrate on model reduction of port-Hamiltonian systems by the Krylov methods (both the Arnoldi and Lanczos procedures) employing the moment matching at infinity. We aim at the port-Hamiltonian structure preservation for the reduced order models which leads to the preservation of the passivity and stability properties. We show that the Krylov methods indeed serve this purpose.

In Section 2 we briefly discuss the Arnoldi method and the Lanczos method as well-known moment matching methods. Basic theory on port-Hamiltonian systems is presented in Section 3, considering both energy and co-energy variable representations for port-Hamiltonian systems.

In Section 4 we show how to obtain the reduced order port-Hamiltonian models using the Arnoldi method employing the projection maps constructed on the basis of the partial reachability and partial observability subspaces in both energy and co-energy coordinates. We define a subclass of port-Hamiltonian systems, characterized by an algebraic condition, for which the Arnoldi method leads to the same reduced order port-Hamiltonian model in the sense of sharing the same transfer function, independently of the subspace that is chosen for the construction of the projection maps.

In Section 5 we exploit the Lanczos method for structure preserving model reduction of port-Hamiltonian systems. We show that under the algebraic condition introduced in Section 4 all the reduced order port-Hamiltonian models obtained in this paper are equivalent, in the sense of sharing the same transfer function. As a result the Arnoldi method preserves $2k$ Markov parameters for the port-Hamiltonian systems from the described subclass. Finally, in Section 6 we present a numerical example of a physical system from this subclass and apply the Arnoldi method to obtain a reduced order model, illustrating how the Arnoldi method preserves not only the port-Hamiltonian structure but also $2k$ Markov parameters.

2. Moment matching for linear systems

In this section we briefly recall the use of the *Krylov methods*, in particular the *Arnoldi method* and the *Lanczos method*, in order to obtain reduced order linear systems preserving the first Markov parameters.

Definition 1 (*Antoulas (2005a)*). The quantities $h_0 = 0, h_k = cA^{k-1}b$, $k > 0$, are the Markov parameters of system (1).

2.1. The Arnoldi method

The idea of the Arnoldi method is to construct a reduced order model by applying a so-called *Galerkin projection* $V_kV_k^T$, $V_k \in \mathbb{R}^{n\times k}$, to a full order linear system (1). The maps $V_k,\ k = 1, \dots, n$, satisfy the following properties:

$$\begin{aligned} &\text{(i) } V_k^TV_k = I_k,\ \text{i.e., the columns of } V_k \\ &\quad \text{are orthonormal,} \\ &\text{(ii) span col } V_k = \text{ span col } \mathcal{R}_k,\ k = 1, 2, \dots, n, \end{aligned} \tag{2}$$

where $\mathcal{R}_k = [b \vdots Ab \vdots \dots \vdots A^{k-1}b] \in \mathbb{R}^{n\times k}$ (which we sometimes also denote by $\mathcal{R}_k(A, b)$) is the partial reachability matrix of the system (1).

In a similar way we can construct the projection maps $W_k \in \mathbb{R}^{n\times k}$, $k = 1, \dots, n$, based on the partial observability matrix, satisfying the following properties:

$$\begin{aligned} &\text{(i) } W_k^TW_k = I_k,\ \text{i.e., the columns of } W_k \\ &\quad \text{are orthonormal,} \\ &\text{(ii) span rows } W_k^T = \text{ span rows } \mathcal{O}_k, \\ &\quad k = 1, 2, \dots, n, \end{aligned} \tag{3}$$

where $\mathcal{O}_k^T = [c^T \vdots (cA)^T \vdots \dots \vdots (cA^{k-1})^T] \in \mathbb{R}^{n\times k}$ is the partial observability matrix of the system (1).

Remark 2. V_k in (2) can be computed by decomposing the partial reachability matrix $\mathcal{R}_k$ using the QR factorization. Similarly, W_k in (3) can be constructed using the LQ factorization of the partial observability matrix $\mathcal{O}_k$. For the details we refer to Antoulas (2005a); Grimme (1997) and the references therein.

Theorem 3. *Let V_k, W_k be matrices satisfying (2) and (3) respectively. Then the kth order systems $\hat{\Sigma}$, $\bar{\Sigma}$*

$$\hat{\Sigma} : \begin{cases} \dot{\hat{x}} = \hat{A}\hat{x} + \hat{b}u, \\ \hat{y} = \hat{c}\hat{x}, \end{cases} \qquad \bar{\Sigma} : \begin{cases} \dot{\bar{x}} = \bar{A}\bar{x} + \bar{b}u, \\ \bar{y} = \bar{c}\bar{x}, \end{cases}$$

where $\hat{A} = V_k^TAV_k$, $\hat{b} = V_k^Tb$, $\hat{c} = cV_k$, $\bar{A} = W_k^TAW_k$, $\bar{b} = W_k^Tb$, $\bar{c} = cW_k$, define reduced order systems with the Markov parameters $\hat{h}_i$, $\bar{h}_i$, $i = 1, \dots, k$, equal to the first k Markov parameters h_i, $i = 1, \dots, k$, of the full order system Σ. Furthermore, $\hat{b}$, $\bar{c}^T$ are multiples of the first basis vector $(1\ 0\ \dots\ 0)^T$.

Proof. The proof for the reduced order system $\hat{\Sigma}$ can be found in Antoulas (2005a). The proof for $\bar{\Sigma}$ is similar; hence omitted. □

2.2. The Lanczos method

In order to apply the *Lanczos method* one has to construct a reduced order model, by applying a so-called *Petrov–Galerkin projection* $V_k W_k^T$, $V_k, W_k \in \mathbb{R}^{n\times k}$, to the full order linear system (1). The maps V_k, W_k satisfy property (ii) of (2) and (3). But in this case V_k, W_k are no longer assumed to be orthonormal but instead biorthogonal (Antoulas, 2005a): $W_k^T V_k = I_k$.

Theorem 4 (Antoulas (2005a)). *Let* $V_k W_k^T, V_k, W_k \in \mathbb{R}^{n\times k}$, *be a Petrov–Galerkin projection. Define the reduced order system* $\tilde{\Sigma}$

$$\begin{cases}\dot{\tilde{x}} = \tilde{A}\tilde{x} + \tilde{b}u,\\ \tilde{y} = \tilde{c}\tilde{x},\end{cases}$$

where $\tilde{A} = W_k^T A V_k$, $\tilde{b} = W_k^T b$, $\tilde{c} = cV_k$. *Then the Markov parameters* $\tilde{h}_i$, $i = 1, \ldots, 2k$, *of* $\tilde{\Sigma}$ *are equal to the first* $2k$ *Markov parameters* h_i, $i = 1, \ldots, 2k$, *of the full order system* Σ. *Furthermore,* $\tilde{A}$ *is tridiagonal and* $\tilde{b}$, $\tilde{c}^T$ *are multiples of the first basis vector* $(1\ 0\ \ldots\ 0)^T$.

The proof is based on the construction of the $k \times k$ Hankel matrix $\mathcal{H}_k$ and its shift $\sigma\mathcal{H}_k$ showing the equality of the $2k$ Markov parameters. Thus the Lanczos method preserves twice as many Markov parameters of the full order model as the Arnoldi method.

3. Linear port-Hamiltonian systems

In the linear case, and in the absence of algebraic constraints, port-Hamiltonian systems take the following form (Polyuga & van der Schaft, 2008; van der Schaft, 2000a)

$$\begin{cases}\dot{x} = (J - R)Qx + bu,\\ y = b^T Qx,\end{cases} \tag{4}$$

with $H(x) = \frac{1}{2}x^T Qx$ the *total energy* (Hamiltonian), $Q = Q^T$ the *energy* matrix and $R = R^T \geqslant 0$ the *dissipation* matrix. The matrices $J = -J^T$ and b specify the *interconnection* structure. Since J is skew-symmetric and R is positive semidefinite it immediately follows that $\frac{d}{dt}\frac{1}{2}x^T Qx = u^T y - x^T QRQx \leqslant u^T y$. Thus if $Q \geqslant 0$ (and the Hamiltonian is non-negative) any port-Hamiltonian system is *passive* (see also Willems (1972) and van der Schaft (2000a)). In this paper we concentrate on the port-Hamiltonian systems with $Q > 0$.

The state variables $x \in \mathbb{R}^n$ are also called *energy* variables, since the total energy $H(x)$ is expressed as a function of these variables. Furthermore, the variables $u \in \mathbb{R}^m$, $y \in \mathbb{R}^m$ are called *power* variables, since their product $u^T y$ equals the power supplied to the system. Physical systems both in the electrical and mechanical domains can be represented as port-Hamiltonian systems. The following example shows the port-Hamiltonian representation of a ladder network in energy coordinates.

Example 1. Consider the linear ladder network in Fig. 1, with $C_1, C_2, L_1, L_2, R_1, R_2$ being the capacitances, inductances and resistances of the corresponding capacitors, inductors and resistors respectively and R_3 the resistance of the load. The port-Hamiltonian representation of this physical system is of the form (4) with

$$J = \begin{bmatrix} 0 & -1 & 0 & 0\\ 1 & 0 & -1 & 0\\ 0 & 1 & 0 & -1\\ 0 & 0 & 1 & 0\end{bmatrix}, \qquad R = \mathrm{diag}\{0, R_1, 0, R_2 + R_3\}, \tag{5}$$

$$Q = \mathrm{diag}\{C_1^{-1}, L_1^{-1}, C_2^{-1}, L_2^{-1}\}, \qquad b^T = [\,1\quad 0\quad 0\quad 0\,],$$

while $x = [\,q_1\quad \phi_1\quad q_2\quad \phi_2\,]^T$ is the state-space vector with q_1, q_2 the charges of the capacitors C_1, C_2 and ϕ_1, ϕ_2 the fluxes of the inductors L_1, L_2 respectively. The input of the system u is given by the current I from the external current source and the output y is the voltage over the first capacitor.

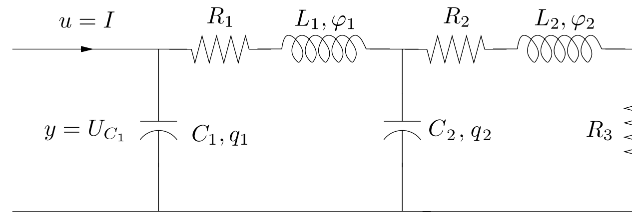

Fig. 1. Ladder network.

We recall from Polyuga and van der Schaft (2008, in press) that a port-Hamiltonian system (4) in so-called co-energy coordinates takes the following form

$$\begin{cases}\dot{e} = Q(J - R)e + Qbu,\\ y = b^T e,\end{cases} \tag{6}$$

which is yet another useful and natural way to look at port-Hamiltonian modeling, especially in the electrical domain, where voltages and currents are used as state-space variables rather than charges and fluxes. The coordinate transformation (Polyuga & van der Schaft, 2008) between energy x and co-energy e coordinates is given by the energy matrix Q:

$$e = Qx. \tag{7}$$

Example 2 (*Continued*). The state-space vector e for the ladder network from Example 1 in the co-energy coordinates is given as $e^T = [\,U_{C_1}\quad I_{L_1}\quad U_{C_1}\quad I_{L_2}\,]$ with U_{C_1}, U_{C_2} the voltages over the capacitors C_1, C_2 and I_{L_1}, I_{L_2} the currents through the inductors L_1, L_2 correspondingly.

4. Reduction of port-Hamiltonian systems by the Arnoldi method

In this section we want to apply the *Arnoldi method* to linear port-Hamiltonian systems.

4.1. Energy coordinates, transforming Q to the identity matrix

Consider a port-Hamiltonian system (4) with $A = (J - R)Q \in \mathbb{R}^{n\times n}$, $b \in \mathbb{R}^n$, $c = b^T Q \in \mathbb{R}^{1\times n}$, $Q > 0$. Then there exists a coordinate transformation S, $x = Sx_I$, such that in the new coordinates

$$Q_I = S^T QS = I. \tag{8}$$

By defining the transformed system matrices as $J_I = S^{-1}JS^{-T}$, $R_I = S^{-1}RS^{-T}$, $b_I = S^{-1}b$, we obtain the transformed port-Hamiltonian system

$$\begin{cases}\dot{x}_I = (J_I - R_I)x_I + b_I u,\\ y = b_I^T x_I\end{cases} \tag{9}$$

with energy $H(x_I) = \frac{1}{2}\|x_I\|^2$.

Theorem 5 (Polyuga and van der Schaft (2009)). *Consider a full order port-Hamiltonian system (9) and construct* V_k, W_k *satisfying (2), (3) respectively using the Arnoldi procedure. Then the kth order reduced systems*

$$\begin{cases}\dot{\hat{x}}_I = (\hat{J}_I - \hat{R}_I)\hat{x}_I + \hat{b}_I u,\\ \hat{y} = \hat{c}_I\hat{x}_I,\end{cases} \tag{10}$$

and

$$\begin{cases} \dot{\bar{x}}_I = (\bar{J}_I - \bar{R}_I)\bar{x}_I + \bar{b}_I u, \\ \bar{y} = \bar{c}_I \bar{x}_I, \end{cases} \tag{11}$$

with the interconnection matrices $\hat{J}_I, \bar{J}_I, \hat{b}_I, \bar{b}_I$, *energy matrices* $\hat{Q}_I, \bar{Q}_I$, *dissipation matrices* $\hat{R}_I, \bar{R}_I$ *and output matrices* $\hat{c}_I, \bar{c}_I$ *given as*

$$\begin{aligned} &\hat{J}_I = V_k^T J_I V_k, \qquad \hat{R}_I = V_k^T R_I V_k, \qquad \hat{Q}_I = I, \\ &\hat{b}_I = V_k^T b_I, \qquad \hat{c}_I = b_I^T V_k, \\ &\bar{J}_I = W_k^T J_I W_k, \qquad \bar{R}_I = W_k^T R_I W_k, \qquad \bar{Q}_I = I, \\ &\bar{b}_I = W_k^T b_I, \qquad \bar{c}_I = b_I^T W_k, \end{aligned}$$

are port-Hamiltonian systems. Furthermore the first k Markov parameters of the reduced order port-Hamiltonian systems (10) and (11) and the full order port-Hamiltonian system (9) are equal:

$$(\hat{h}_I)_i = (\bar{h}_I)_i = (h_I)_i = h_i, \quad i = 1, \dots, k,$$

while $\hat{b}_I, \bar{c}_I^T$ *are multiples of the first basis vector* $(1\ 0\ \dots\ 0)^T$.

Proof. We prove the Theorem for the reduced order model (10); the proof for the reduced order model (11) is analogous. Clearly $\hat{J}_I$ is skew-symmetric and $\hat{R}_I$ is symmetric and positive semidefinite. Moreover $\hat{c}_I = \hat{b}_I^T \hat{Q}_I$. Therefore the reduced order model (10) is port-Hamiltonian. The equality of the first k Markov parameters $(\hat{h}_I)_i = (h_I)_i$ and the proportionality of $\hat{b}_I$ to the first basis vector follow directly from Theorem 3. The equality $(h_I)_i = h_i$ is due to the fact that the Markov parameters are invariant under state-space coordinate transformations. □

Remark 6. Note that there are many ways to compute the coordinate transformation S in (8). One of them is by means of the computationally efficient Cholesky factorization (see Golub and Van Loan (1996)) of the matrix Q.

4.2. Co-energy coordinates, transforming Q to the identity matrix

The similarity coordinate transformation T,

$$e = Te_I, \tag{12}$$

which transforms Q to the identity, can be applied to a port-Hamiltonian system in co-energy coordinates (6) resulting in the transformed port-Hamiltonian system

$$\begin{cases} \dot{e}_I = (J_I - R_I)e_I + b_I u, \\ y = b_I^T e_I \end{cases} \tag{13}$$

with system matrices in general different from those in (9). For the sake of simplicity we keep the notation alike.

Using the projection maps V_k, W_k (which are in general different from those in energy coordinates) in a similar fashion as in Theorem 5, we obtain the kth order reduced port-Hamiltonian system

$$\begin{cases} \dot{\hat{e}}_I = (\hat{J}_I - \hat{R}_I)\hat{e}_I + \hat{b}_I u, \\ \hat{y} = \hat{c}_I \hat{e}_I \end{cases} \tag{14}$$

in case of V_k, and the port-Hamiltonian system

$$\begin{cases} \dot{\bar{e}}_I = (\bar{J}_I - \bar{R}_I)\bar{e}_I + \bar{b}_I u, \\ \bar{y} = \bar{c}_I \bar{e}_I \end{cases} \tag{15}$$

in case of W_k, which both preserve the first k Markov parameters having $\hat{b}_I, \bar{c}_I^T$ equal to the first basis vector $(1\ 0\ \dots\ 0)^T$.

4.3. General Q

The partial reachability matrix for the port-Hamiltonian system in co-energy coordinates (6) is given as

$$\mathcal{R}_k = [Qb \vdots QFQb \vdots \dots \vdots (QF)^{k-1}Qb] \in \mathbb{R}^{n\times k}, \tag{16}$$

where $F = J - R$. We will use this notation throughout the paper.

Having taken the Q matrix to the left side we obtain

$$\begin{cases} Q^{-1}\dot{e} = (J - R)e + bu, \\ y = b^T e. \end{cases} \tag{17}$$

Theorem 7. *Consider a full order port-Hamiltonian system (17) and construct* V_k *satisfying (2) using the Arnoldi procedure. Then the kth order reduced system*

$$\begin{cases} \hat{Q}^{-1}\dot{\hat{e}} = (\hat{J} - \hat{R})\hat{e} + \hat{b}u, \\ \hat{y} = \hat{c}\hat{e} \end{cases} \tag{18}$$

with the interconnection matrices $\hat{J}_I, \hat{b}_I$, *energy matrix* $\hat{Q}_I$, *dissipation matrix* $\hat{R}_I$ *and output matrix* $\hat{c}_I$ *given as*

$$\begin{aligned} &\hat{J} = V_k^T J V_k, \qquad \hat{R} = V_k^T R V_k, \qquad \hat{F} = \hat{J} - \hat{R} = V_k^T F V_k, \\ &\hat{Q}^{-1} = V_k^T Q^{-1} V_k, \qquad \hat{b} = V_k^T b, \qquad \hat{c} = b^T V_k, \end{aligned}$$

is a port-Hamiltonian system. Furthermore the first k Markov parameters of the reduced order port-Hamiltonian system (18) and of the full order port-Hamiltonian system (17) are equal:

$$\hat{h}_i = h_i, \quad i = 1, \dots, k.$$

Proof. Firstly, let us show that $V_k\hat{Q}V_k^TQ^{-1}$ is the identity mapping on $\mathcal{R}_k$ given in (16), i.e.,

$$V_k\hat{Q}V_k^TQ^{-1}\mathcal{R}_k = \mathcal{R}_k. \tag{19}$$

Indeed, having observed that $\mathcal{R}_k$ can be decomposed as $\mathcal{R}_k = V_kU$ with U upper triangular (for details see Remark 2), we have $V_k\hat{Q}V_k^TQ^{-1}\mathcal{R}_k = V_k\hat{Q}V_k^TQ^{-1}V_kU = V_k\hat{Q}\hat{Q}^{-1}U = \mathcal{R}_k$. Secondly, let us prove that

$$\hat{Q}V_k^TQ^{-1}\mathcal{R}_k = \hat{\mathcal{R}}_k, \tag{20}$$

where $\hat{\mathcal{R}}_k$ is the partial reachability matrix of the reduced order system (18):

$$\hat{\mathcal{R}}_k = [\hat{Q}\hat{b} \vdots \hat{Q}\hat{F}\hat{Q}\hat{b} \vdots \dots \vdots (\hat{Q}\hat{F})^{k-1}\hat{Q}\hat{b}].$$

It follows that

$$\begin{aligned} \hat{Q}V_k^TQ^{-1}Qb &= \hat{Q}V_k^Tb = \hat{Q}\hat{b}, \\ \hat{Q}V_k^TQ^{-1}QFQb &= \hat{Q}V_k^TFQb \\ &= \hat{Q}V_k^TF(V_k\hat{Q}V_k^TQ^{-1}(Qb)) \\ &= \hat{Q}(V_k^TFV_k)\hat{Q}(V_k^Tb) \\ &= \hat{Q}\hat{F}\hat{Q}\hat{b}, \\ &\ \vdots \\ \hat{Q}V_k^TQ^{-1}(QF)^{k-1}Qb &= \dots \\ &= (\hat{Q}\hat{F})^{k-1}\hat{Q}\hat{b}, \end{aligned}$$

using (19) and the induction principle.

Since the Markov parameters $(h_1 \dots h_k)$ are related to the partial reachability matrix $\mathcal{R}_k$ as $(h_1 \dots h_k) = c\mathcal{R}_k$ (see Antoulas (2005a)), we can use (19) and (20) to prove the equality of the Markov parameters of the full and reduced order port-Hamiltonian systems:

$$(\hat{h}_1 \dots \hat{h}_k) = \hat{c}\hat{\mathcal{R}}_k = cV_k\hat{Q}V_k^TQ^{-1}\mathcal{R}_k = c\mathcal{R}_k = (h_1 \dots h_k). \quad \square$$

4.4. Equivalence of the reduced order models

The full order models in (4), (6), (9) and (13) share the same transfer function since Q in (7), S in (8) and T in (12) are nonsingular coordinate transformations. The following Theorem shows that the reduced order systems obtained in different coordinates using the projections maps based on the partial reachability subspaces are equivalent, in the sense of sharing the same transfer function.

Theorem 8. *The reduced order port-Hamiltonian models obtained by the Arnoldi method using the projection maps V_k based on the partial reachability subspaces in energy coordinates (10) and co-energy coordinates (14) and (18) have the same transfer function.*

Proof. First we prove the equivalence of the reduced order models in (10) and (14). The transfer function of (10) is given as

$$\begin{aligned} G_x(s) &= b_x^T V_x(sI - V_x^T F_x V_x)^{-1} V_x^T b_x \\ &= b^T S^{-T} V_x(sI - V_x^T S^{-1} F S^{-T} V_x)^{-1} V_x^T S^{-1} b \\ &= b^T \bar{V}_x(sI - \bar{V}_x^T F \bar{V}_x)^{-1} \bar{V}_x^T b, \end{aligned}$$

where $V_x := V_k, F_x := J_I - R_I, b_x := b_I$ are from (10) in order to distinguish these matrices from $V_e := V_k, F_e := J_I - R_I, b_e := b_I$ from (14), and $\bar{V}_x := S^{-T} V_x$. At the same time the transfer function of (14) is given as

$$\begin{aligned} G_e(s) &= b_e^T V_e(sI - V_e^T F_e V_e)^{-1} V_e^T b_e \\ &= b^T T V_e(sI - V_e^T T^T F T V_e)^{-1} V_e^T T^T b \\ &= b^T \bar{V}_e(sI - \bar{V}_e^T F \bar{V}_e)^{-1} \bar{V}_e^T b, \end{aligned}$$

where $\bar{V}_e := T V_e$. Therefore if the columns of $\bar{V}_x$ and $\bar{V}_e$ span the same subspace then the transfer functions $G_x(s)$ and $G_e(s)$ will be equal. Since

$$\begin{aligned} \text{span col } V_x &= \text{span col } \mathcal{R}_k(F_x, b_x) \\ &= \text{span col } [b_x \vdots F_x b_x \vdots \dots \vdots F_x^{k-1} b_x] \\ &= \text{span col } [S^{-1} b \vdots \dots \vdots (S^{-1} F S^{-T})^{k-1} S^{-1} b] \\ &= S^{-1} \text{span col } [b \vdots \dots \vdots (FQ)^{k-1} b] \\ &= S^{-1} \text{span col } \mathcal{R}_k(FQ, b), \end{aligned}$$

and, similarly,

$$\text{span col } V_e = T^T \text{span col } \mathcal{R}_k(FQ, b),$$

it follows (using $Q = S^{-T} S^{-1}, Q = T T^T$) that

$$\begin{aligned} \text{span col } \bar{V}_x &= S^{-T} \text{span col } V_x \\ &= S^{-T} S^{-1} \text{span col } \mathcal{R}_k(FQ, b) \\ &= Q \text{ span col } \mathcal{R}_k(FQ, b), \end{aligned}$$

$$\text{span col } \bar{V}_e = Q \text{ span col } \mathcal{R}_k(FQ, b).$$

And thus indeed span col $\bar{V}_x$ = span col $\bar{V}_e$. Hence $G_e(s) = G_x(s)$, which shows the equivalence of the reduced order models in (10) and (14). It is easy to see that the reduced order model in (18) is obtained using V_k based on the same partial reachability subspace as $\bar{V}_x$, namely,

$$\begin{aligned} \text{span col } V_k &= \text{span col } \mathcal{R}_k(QF, Qb) \\ &= Q \text{ span col } \mathcal{R}_k(FQ, b). \end{aligned}$$

Therefore (18) is equivalent to the reduced order models in (10) and (14), which completes the proof. □

In a similar way it is possible to show the equivalence between the reduced order port-Hamiltonian models (11) and (15) obtained using the projection maps W_k based on the partial observability subspaces.

In general, the reduced order models obtained by applying the Arnoldi method using the partial reachability matrix and the partial observability matrix are *not* equivalent. Nevertheless, under the condition stated in the following Theorem we can prove that these reduced order models indeed are equivalent, sharing the same transfer function.

Theorem 9. *The reduced order port-Hamiltonian model (10) obtained using the projection map V_k based on the partial reachability matrix $\mathcal{R}_k(A_I, b_I)$ and the reduced order port-Hamiltonian model (11) obtained using the projection map W_k based on the partial observability matrix $\mathcal{R}_k(A_I^T, c_I^T)$ share the same transfer function if the following condition is satisfied:*

$$\text{span col } \mathcal{R}_k(FQ, b) = \text{span col } \mathcal{R}_k(F^T Q, b). \quad (21)$$

Proof. The proof is similar to the proof of Theorem 8, hence omitted. □

Purely undamped port-Hamiltonian systems with $R = 0, F = J$ and therefore $F^T = -F$, and purely damped port-Hamiltonian systems with $J = 0, F = R$ and $F^T = F$ obviously satisfy condition (21). For the details on purely undamped and purely damped port-Hamiltonian systems see Polyuga and van der Schaft (2008, in press). However, in many other cases condition (21) is satisfied as well, as shown by the following example.

Example 3 (*Continued*). Consider system (5) with unit values of the capacitors C_1, C_2 and inductors L_1, L_2, while $R_1 = R_2 = 0.2$, $R_3 = 0.4$. Then

$$\mathcal{R}_2(FQ, b) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \qquad \mathcal{R}_3(FQ, b) = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -0.2 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix},$$

$$\mathcal{R}_2(F^T Q, b) = \begin{bmatrix} 1 & 0 \\ 0 & -1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \qquad \mathcal{R}_3(F^T Q, b) = \begin{bmatrix} 1 & 0 & -1 \\ 0 & -1 & 0.2 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}.$$

One can easily see that for $i = 2, 3$

$$\text{span col } \mathcal{R}_i(FQ, b) = \text{span col } \mathcal{R}_i(F^T Q, b).$$

An example of a port-Hamiltonian system not satisfying condition (21) is the following.

Example 4. Consider a four-dimensional port-Hamiltonian system with

$$J = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & -2 & 0 \\ 0 & 2 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \qquad R = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$b = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}^T,$$

and $Q = I$. Then

$$\mathcal{R}_3(FQ, b) = \begin{bmatrix} 0 & -1 & 2 \\ 1 & -1 & -3 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix},$$

$$\mathcal{R}_3(F^TQ, b) = \begin{bmatrix} 0 & 1 & -2 \\ 1 & -1 & -3 \\ 0 & -3 & 6 \\ 0 & 0 & 3 \end{bmatrix},$$

and thus

$$\text{span col } \mathcal{R}_3(FQ, b) \neq \text{span col} \mathcal{R}_3(F^TQ, b).$$

For this system the transfer functions $G_V(s)$, $G_W(s)$ of the three-dimensional reduced order port-Hamiltonian models obtained using projections maps V_k, W_k are given by the following different expressions:

$$G_V(s) = \frac{s^2 + s + 0.5}{s^3 + 2s^2 + 5.5s + 0.5},$$
$$G_W(s) = \frac{s^2 + s + 0.9}{s^3 + 2s^2 + 5.9s + 0.9}.$$

5. Reduction of port-Hamiltonian systems by the Lanczos method

According to Theorem 4 the Lanczos method preserves $2k$ Markov parameters of the full order system. On the other hand the port-Hamiltonian structure in general is not preserved. Indeed, a reduced order model would take the following form $\tilde{A} = W_k(J - R)QV_k$, $\tilde{b} = W_k b$, $\tilde{c} = b^T QV_k$, which is in general not port-Hamiltonian. In this section we show how to construct the projection maps V_k, W_k in such a way that the Lanczos method preserves the port-Hamiltonian structure for systems satisfying a specific condition, which is, in fact, precisely the algebraic condition given in (21).

Theorem 10. *Consider a full order port-Hamiltonian system (4) and construct V_k satisfying property* (ii) *of (2) such that $V_k^T QV_k = I_k$. Then the kth order reduced system*

$$\begin{cases} \dot{\tilde{x}} = (\tilde{J} - \tilde{R})\tilde{x} + \tilde{b}u, \\ \tilde{y} = \tilde{c}\tilde{x} \end{cases} \tag{22}$$

with the interconnection matrices $\tilde{J}_l$, $\tilde{b}_l$, energy matrix $\tilde{Q}_l$, dissipation matrix $\tilde{R}_l$ and output matrix $\tilde{c}_l$ given as

$$\tilde{J} = V_k^T QJQV_k, \qquad \tilde{R} = V_k^T QRQV_k, \qquad \tilde{Q} = I,$$
$$\tilde{b} = V_k^T Qb, \qquad \tilde{c} = b^T QV_k,$$

is a port-Hamiltonian system reduced by the Lanczos method with $W_k = QV_k$ if condition (21) holds true. Furthermore the first $2k$ Markov parameters of the reduced order port-Hamiltonian system (22) and the full order port-Hamiltonian system (4) are equal:

$$\tilde{h}_i = h_i, \quad i = 1, \ldots, 2k.$$

Moreover $\tilde{A}$ is tridiagonal and $\tilde{b}$ and $\tilde{c}^T$ are multiples of the first basis vector $(1\ 0\ \ldots\ 0)^T$.

Proof. In this case the subspaces spanned by the columns of V_k, W_k can be represented as follows:

$$\begin{aligned} \text{span col } V_k &= \text{span col } \mathcal{R}_k(FQ, b); \\ \text{span col } W_k &= \text{span col } \mathcal{R}_k(A^T, c^T) \\ &= \text{span col } \mathcal{R}_k(QF^T, Qb) \\ &= Q \text{ span col } \mathcal{R}_k(F^TQ, b), \end{aligned}$$

with V_k, W_k as in Theorem 4. Then condition (21) implies that

$$\text{span col } W_k = \text{span col } QV_k.$$

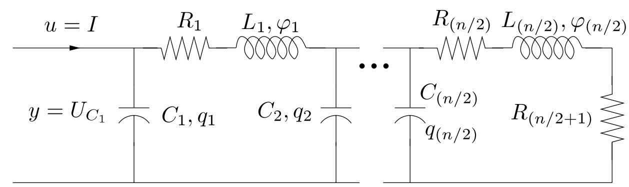

Fig. 2. n-dimensional ladder network.

Therefore one can choose any W_k such that its columns span the same subspace as the columns of QV_k. In particular, taking W_k as QV_k preserves the port-Hamiltonian structure for the reduced order model. Preservation of $2k$ Markov parameters and the fact that $\tilde{A}$ is tridiagonal and $\tilde{b}$ and $\tilde{c}^T$ are multiples of the first basis vector $(1\ 0\ \ldots\ 0)^T$ follow directly from Theorem 4, completing the proof. □

We can also apply the Lanczos method in co-energy coordinates and show that the reduced order model in co-energy coordinates shares the same transfer function with the reduced order model in (22) following the discussion from the previous section.

The next result establishes a relation between the reduced order port-Hamiltonian models obtained by both the Arnoldi and the Lanczos methods provided that condition (21) is satisfied.

Theorem 11. *The reduced order port-Hamiltonian model (10) in energy coordinates obtained by the Arnoldi method and the reduced order port-Hamiltonian model (22) in energy coordinates obtained by the Lanczos method share the same transfer function if condition (21) is satisfied. Furthermore, under condition (21) all the reduced order port-Hamiltonian models (10), (11), (14), (15), (18), obtained by the Arnoldi method, and the reduced order port-Hamiltonian model (22), obtained by the Lanczos method, both in energy and co-energy coordinates are equivalent, sharing the same transfer function.*

Proof. The proof is similar to the proof of Theorem 8, hence omitted. □

Corollary 12. *If (21) is satisfied then a reduced order port-Hamiltonian model obtained by the Arnoldi method matches $2k$ Markov parameters.*

Remark 13. Note that the projected systems (10), (11), (14), (15), (18) and (22) are automatically passive since they inherit the port-Hamiltonian structure of the full order systems with $\hat{Q}$ being positive definite. See also van der Schaft (2000a) and Willems (1972).

Remark 14. The Arnoldi and Lanczos methods for general asymptotically stable linear systems sometimes lead to unstable reduced order models. One way to overcome this problem is to use implicitly restarted Krylov methods, as discussed in Antoulas (2005a) and Lehoucq (1995), and the references therein. For port-Hamiltonian systems with $Q > 0$ a reduced order port-Hamiltonian model can loose its asymptotic stability, but can never be unstable. This follows from the passivity property of the reduced order port-Hamiltonian systems.

6. Numerical example

Consider the ladder network in Fig. 2, which is an extension of the ladder network in Example 1. We take the current I as the input and the voltage over the first capacitor U_{C_1} as the port-Hamiltonian output. The state variables are as follows: x_1 is the charge q_1 of C_1, x_2 is the flux ϕ_1 of L_1, x_3 is the charge q_2 of C_2, x_4 is the flux ϕ_2 of L_2, etc.

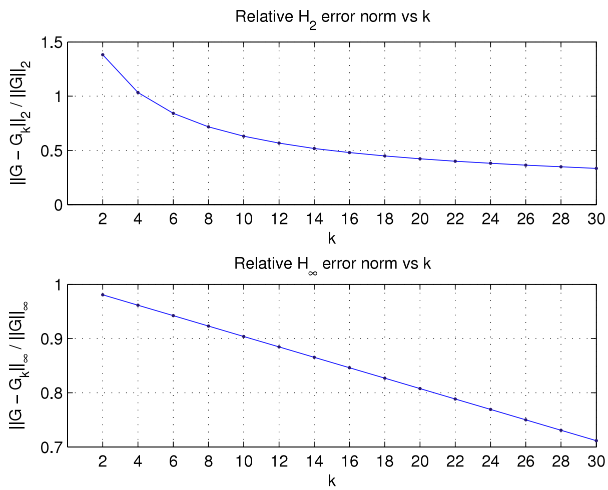

Fig. 3. Evolution of the relative $\mathcal{H}_2$ and $\mathcal{H}_\infty$ norms.

A minimal realization of this port-Hamiltonian ladder network for the order $n = 6$ is

$$A = \begin{bmatrix} 0 & -\frac{1}{L_1} & 0 & 0 & 0 & 0 \\ \frac{1}{C_1} & -\frac{R_1}{L_1} & -\frac{1}{C_2} & 0 & 0 & 0 \\ 0 & \frac{1}{L_1} & 0 & -\frac{1}{L_2} & 0 & 0 \\ 0 & 0 & \frac{1}{C_2} & -\frac{R_2}{L_2} & -\frac{1}{C_3} & 0 \\ 0 & 0 & 0 & \frac{1}{L_2} & 0 & -\frac{1}{L_3} \\ 0 & 0 & 0 & 0 & \frac{1}{C_3} & -\frac{R_3+R_4}{L_3} \end{bmatrix},$$

$$b = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T,$$

$$c = \begin{bmatrix} \frac{1}{C_1} & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

where $A = (J - R)Q$ with $R = \mathrm{diag}\{0, R_1, 0, R_2, 0, R_3 + R_4\}$ and J, Q of the same structure as in (5).

Adding another LC pair to the network (with appropriate resistors), which would correspond to an increase of the dimension of the model by two, will modify the ABC-model in the following way. The sub-diagonal of the matrix A will contain additionally $L_{n/2-1}^{-1}, C_{n/2}^{-1}$. The superdiagonal of A will contain $-C_{n/2}^{-1}, -L_{n/2}^{-1}$. Furthermore, the main diagonal of A will have $-\frac{R_{n/2-1}}{L_{n/2-1}}$ in the $(n-2, n-2)$ position, zero in the $(n-1, n-1)$ position and $-\frac{R_{n/2}+R_{n/2+1}}{L_{n/2}}$ in the (n, n) position. b and c matrices will obtain zeros in the appropriate positions.

We considered the 100-dimensional full order port-Hamiltonian network with unit values of the capacitors C_i and inductors L_i, while $R_i = 0.2$, $i = 1, \ldots, 50$, $R_{51} = 0.4$. We applied the Arnoldi method to obtain the reduced order port-Hamiltonian model (10), as shown in Theorem 5. The reduced order systems are constructed for the orders $k = 2$ to $k = 30$ with increments of 2. Evolution of the relative $\mathcal{H}_2$ and $\mathcal{H}_\infty$ norms is shown in Fig. 3. As expected, both $\mathcal{H}_2$ and $\mathcal{H}_\infty$ relative norms decay as the dimension k of a reduced order system increases. Reduced order systems inherit the port-Hamiltonian structure, are asymptotically stable and passive.

The Amplitude Bode plots of the full, reduced and error systems for $k = 20$ are shown in Fig. 4. The figure exhibits that the $\mathcal{H}_2$ and $\mathcal{H}_\infty$ error norms are accumulated for small frequencies and that the distance between the frequency responses of the full and reduced order systems decays as frequency increases since we match moments at infinity.

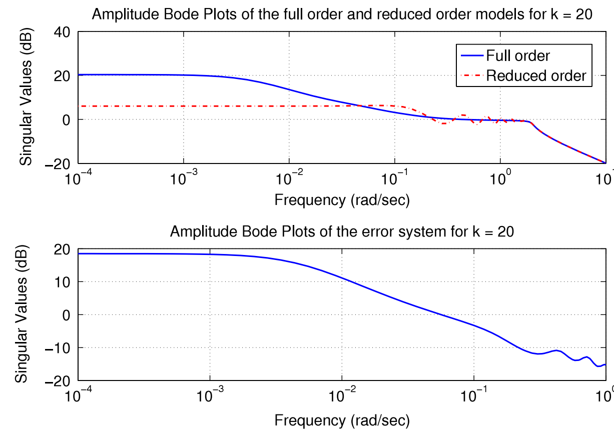

Fig. 4. Amplitude Bode plots for $k = 20$.

In fact, for the port-Hamiltonian system considered here condition (21) is satisfied as was already discussed in Example 3. Indeed, for $k = 4$ the reduced order matrices are

$$\hat{A} = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & -0.2 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -0.2 \end{bmatrix}, \quad \hat{b} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad \hat{c} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}^T,$$

$\hat{b}, \hat{c}^T$ are the multiples of the first unit vector and $\hat{A}$ is tridiagonal. Therefore even though the reduced order port-Hamiltonian model is obtained using the Arnoldi method as shown in Theorem 5, it is equivalent to that of the Lanczos method as Theorem 11 explains. Moreover, due to Corollary 12 the reduced order port-Hamiltonian model preserves $2k$ Markov parameters which can be readily checked for this particular case: $(h_1 \;\; \ldots \;\; h_{2k}) = (1 \;\; 0 \;\; -1 \;\; 0.2 \;\; 1.96 \;\; -0.7920 \;\; -4.7616 \;\; 2.9363) = (\hat{h}_1 \;\; \ldots \;\; \hat{h}_{2k})$.

7. Epilogue

In this paper we applied the Krylov methods in order to reduce a full order port-Hamiltonian system to a reduced order system which inherits the port-Hamiltonian structure. In particular, we showed how the Arnoldi method, which preserves k Markov parameters, can be employed for this purpose in energy and co-energy coordinates using the projection maps constructed both on the partial reachability and partial observability subspaces. We showed the equivalence of the reduced order models in the sense of sharing the same transfer function.

We exploited the Lanczos method, which preserves $2k$ Markov parameters, for structure preserving model reduction of a subclass of port-Hamiltonian systems, characterized by an algebraic condition. For this subclass the Lanczos method is proven to produce a reduced order port-Hamiltonian model which is equivalent to that of the Arnoldi method. Therefore the Arnoldi method applied to a port-Hamiltonian system from the subclass preserves twice as many Markov parameters as it does for a general linear system.

Both methods considered preserve the port-Hamiltonian structure, implying, among others, the passivity property.

Questions concerning general error bounds for the structure preserving port-Hamiltonian model reduction methods, numerical efficiency and the physical realization of the obtained port-Hamiltonian reduced order models as well as the further characterization of the subclasses of port-Hamiltonian systems are currently under investigation.

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References

Antoulas, A. C. (2005a). *Approximation of large-scale dynamical systems*. Philadelphia: SIAM.

Antoulas, A. C. (2005b). A new result on passivity preserving model reduction. *Systems & Control Letters*, *54*(4), 361–374.

Gallivan, K., Grimme, E., & Van Dooren, P. M. (1999). Model reduction of large-scale systems: Rational Krylov versus balancing techniques. In H. Bulgak, & C. Zenger (Eds.), *Error control and adaptivity in scientific computing*: *Vol. 536* (pp. 177–190).

Golub, G. H., & Van Loan, C. F. (1996). *Matrix computations* (3rd ed.). Johns Hopkins.

Gragg, W. B., & Lindquist, A. (1983). On the partial realization problem. *Linear Algebra and its Applications*, *50*, 277–319.

Grimme, E. (1997). *Krylov projection methods for model reduction. Ph.D. thesis*. Coordinated Science Laboratory, University of Illinois at Urbana-Champaign.

Grimme, E., Gallivan, K., & Van Dooren, P. M. (1998). On some recent developments in projection-based model reduction. *ENUMATH-97* (pp. 98–113).

Gugercin, S., Antoulas, A. C., & Beattie, C. A. (2008). $\mathcal{H}_2$ model reduction for large-scale linear dynamical systems. *SIAM Journal on Matrix Analysis and Applications*, *30*(2), 609–638.

Gutknecht, M. H., et al. (1994). The Lanczos process and Padé approximation. In J. D. Brown (Ed.)., *Proceedings of the cornelius lanczos international centenary conference* (pp. 61–75). Philadelphia: SIAM.

Hartmann, C. (2009). Balancing of dissipative Hamiltonian systems. In I. Troch, & F. Breitenecker (Eds.), *ARGESIM-Reports*: *Vol. 35*. *Proceedings of MATHMOD 2009*, Vienna, February 11–13, (pp. 1244–1255). Vienna, Austria: Vienna Univ. of Technology.

Hartmann, C., Vulcanov, V. -M., & Schütte, Ch. (2010). Balanced truncation of linear second-order systems: A Hamiltonian approach. *Multiscale Modelling and Simulation* (in press). Available from http://proteomics-berlin.de/28/.

Ionutiu, R., Rommes, J., & Antoulas, A. C. (2008). Passivity-preserving model reduction using dominant spectral-zero interpolation. *IEEE Transactions on Computer-Aided Design*, *27*(12), 2250–2263.

Komzsik, L. (2003). *The Lanczos method: Evolution and application*. Philadelphia: SIAM.

Lall, S., Krysl, P., & Marsden, J. E. (2003). Structure-preserving model reduction for mechanical systems. *Physica D: Nonlinear Phenomena*, *184*, 304–318.

Lehoucq, R. B. (1995). *Analysis and implementation of an implicitly restarted arnoldi iteration. Ph.D. thesis*. Rice University.

Mehrmann, V., & Xu, H. (2000). Numerical methods in control. *Journal of Computational and Applied Mathematics*, *123*, 371–394.

Moore, B. C. (1981). Principal component analysis in linear systems. *IEEE Transactions on Automatic Control*, *AC-26*, 17–32.

Ortega, R., van der Schaft, A. J., Mareels, I., & Maschke, B. M. (2001). Putting energy back in control. *Control Systems Magazine*, *21*, 18–33.

Polyuga, R. V., & van der Schaft, A. J. (2008). Structure preserving model reduction of port-Hamiltonian systems. In *Proceedings of the 18th international symposium on mathematical theory of networks and systems*, Blacksburg, Virginia, USA, July 28–August 1.

Polyuga, R. V., & van der Schaft, A. J. (2009). Moment matching for linear port-Hamiltonian systems. In *Proceedings of the European control conference 2009* (pp. 4715–4720) Budapest, Hungary, August 23–26.

Polyuga, R. V., & van der Schaft, A. J. (2010). Structure preserving port-Hamiltonian model reduction of electrical circuits. In P. Benner, M. Hinze, and J. ter Maten (Eds.), *Lecture notes in electrical engineering. Model reduction for circuit simulation*. Berlin/Heidelberg: Springer-Verlag (in press).

Schilders, W. H. A., van der Vorst, H. A., & Rommes, J. (2008). *ECMI series on mathematics in industry*: *Vol. 13*. *Model order reduction: Theory, research aspects and applications*. Berlin, Heidelberg: Springer-Verlag.

Sorensen, D. C. (2005). Passivity preserving model reduction via interpolation of spectral zeros. *Systems & Control Letters*, *54*(4), 347–360.

van der Schaft, A. J. (2000a). *L2-gain and passivity techniques in nonlinear control*. Springer-Verlag.

van der Schaft, A. J. (2000b). Port-controlled Hamiltonian systems: Towards a theory for control and design of nonlinear physical systems. *Journal of the Society of Instrument and Control Engineers of Japan (SICE)*, *39*, 91–98.

van der Schaft, A. J., & Maschke, B. M. (1995). The Hamiltonian formulation of energy conserving physical systems with external ports. *Archiv für Elektronik und Übertragungstechnik*, *49*(5/6), 362–371.

Van Dooren, P. M. (1995). The Lanczos algorithm and Padé approximations. In *Benelux meeting on systems and control*, Houthalen, Belgium.

Willems, J. C. (1972). Dissipative dynamical systems. *Archive for Rational Mechanics and Analysis*, *45*, 321–393.

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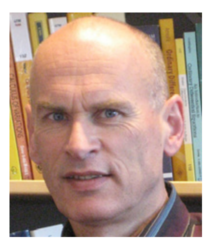

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